lemmas play a crucial role.

Lemma 1. *Let g be an element of G and let $\{W_1, W_2, \ldots, W_m\}$ be a covering of G by evenly covered neighborhoods such that $g \in W_1 \setminus \bigcup_{k=2}^{m} W_k$. Let $p^{-1}(W_1) = \bigsqcup_{l=1}^{n} V_l$ and $p^{-1}(g) \cap V_1 = \{y_0\}$. Then there exists a neighborhood $V_0 \subset V_1$ of the point y_0 satisfying the following property: if $T_{t_0}(V_0) \cap V_0 \neq \emptyset$ for some $t_0 \in R$, then $T_{t_0}(V_0) \subset V_1$.*

Lemma 2. *An orbit $R_x = \{T_t(x) : t \in R\}$ of each point $x \in X$ is dense in the connected space X.*

Finally, we formulate the theorem on covering groups for compact solenoids.

Theorem. *Let $p : X \to G$ be an n-fold covering of compact solenoidal group G by a connected topological space X. Then there exists a group structure in X turning $p : X \to G$ into a homomorphism between compact abelian groups.*

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EXTREME PROPERTIES OF THE TAYLOR-SAFFMAN CURVE

Since the seminal Taylor-Saffman experiments and theoretical investigations of viscous fingers in the Hele-Shaw apparatus many attempts have been done to explain the mechanisms of so-called "selection". Still, it is unclear why practically a half-channel finger appears though mathematically a whole family of interfaces is possible. We apply the method of inverse boundary-value problems [1] and arrive at the Taylor-Saffman shapes from solution of optimal shape design problems [2,3] illustrating that "selected" curves possess fascinating extreme properties. In particular, we show that:

a) The Taylor-Saffman bubble is a limiting case of the Taylor-Saffman finger.

b) The Taylor-Saffman bubble coincides with the Polubarinova-Kochina subsurface contour of a concrete dam of constant hydraulic gradient, which is a solution of an isoperimetric problem on maximum of the cross-sectional area of the dam.

c) The Taylor-Saffman half-channel finger coincides with the Morse-Feshbach [4] "extreme" boundary of a variable condenser. Hence, the finger is an equipotential line generated by a semi-infinite linear source.

d) The Taylor-Saffman finger coincides with an abrupt interface between seeping fresh and stagnant saline water in a polder-type system [5], i.e., the finger is a stream line of a vortex generated flow.

Obtained explicit analytic solutions of optimization problems are also used for estimations of integral and local flow characteristics (total flow rate and field intensity).

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ON PROJECTIVE MODULES OF FINITE DUAL GOLDI DIMENSION

Let R be an associative ring with unit, P will be a left unital R-module and $J(P)$ denote the Jacobson's radical of P.

A submodule N of P is said *small* in P if for every submodule U of P the equation $N + U = P$ involves $U = P$. A module P is said to be hollow if $P \neq 0$ and every proper submodule of P is small in P. A module P is said to have *finite hollow dimension* (or *finite dual Goldie dimension*) if there exists an exact sequence

$$P \xrightarrow{g} \bigoplus_{i=1}^{n} H_i \longrightarrow 0$$

where all H_i are hollow and the kernel of $R-$homomorphism g is small in P. Then n is called the hollow dimension of P and we write $hdim(P) = n$.

In the paper [1] it was formulated the question: *Is every projective R-module P with semilocal endomorphism ring $Hom_R(P, P)$ finitely generated?*

If P is projective R-module and $Hom_R(P, P)$ is semilocal ring then by the theorem 3.10 [1] we have $hdim_R(P) < \infty$. We have proved:

Theorem *Let P be a projective R-module and $Hom_R(P, P)$ is semilocal ring. Then the following conditions are equivalent:*
(a) P is finitely generated.
(b) $hdim_R(P) = hdim_R(P/J(P))$.

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